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One-loop renormalisation of $N = \frac{1}{2}$ supersymmetric gauge theory

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We show that $N = \frac{1}{2}$ supersymmetric gauge theory is renormalisable at one loop, but only after gauge invariance is restored in a non-trivial fashion.

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1. Introduction

There has recently been much interest in theories defined on non-anti-commutative superspace [1] [2]. Such theories are non-hermitian and turn out to have only half the supersymmetry of the corresponding ordinary supersymmetric theory–hence the term "$N = \frac{1}{2}$ supersymmetry". These theories are not power-counting renormalisable but it has been argued[3] –[6] that they are in fact nevertheless renormalisable, in the sense that only a finite number of additional terms need to be added to the lagrangian to absorb divergences to all orders. This is primarily because although the theory contains operators of dimension five and higher, they are not accompanied by their hermitian conjugates which would be required to generate divergent diagrams. This argument does not of course guarantee that the precise form of the lagrangian will be preserved by renormalisation; nor does the $N = \frac{1}{2}$ supersymmetry, since some terms in the lagrangian are inert under this symmetry. Moreover, the argument also requires (in the gauged case) the assumption of gauge invariance to rule out some classes of divergent structure. We shall show here by explicit calculation that there are problems with this assumption; even at one loop, at least in the standard class of gauges, divergent non-gauge-invariant terms are generated. However, in the case of pure $N = \frac{1}{2}$ supersymmetry (i.e. no chiral matter) we shall display a divergent field redefinition which miraculously removes the non-gauge-invariant terms and restores gauge invariance. The form of the lagrangian is not quite preserved by renormalisation. However, a slightly modified version of the original lagrangian (which is still $N = \frac{1}{2}$ supersymmetric) does have a form preserved under renormalisation. We shall try to give sufficient details of our calculations in order to enable the interested reader to check them. The one-loop calculation of the divergences in pure $N = \frac{1}{2}$ gauge theory has been performed before [7] but we disagree with this result as we shall explain. The original non-anticommutative theory defined in superfields appears to require a $U(N)$ gauge group[2][3]. However the component form of the action is easily adapted to $SU(N)$, as we shall see; and we shall argue that in addition to being simpler for calculational purposes, it is only for $SU(N)$ that a form-invariant lagrangian can be defined. We shall therefore assume the gauge group $SU(N)$ so that the basic commutation relations are (for the fundamental representation):

$$[R^a, R^b] = if^{abc}R^c, \quad \{R^a, R^b\} = d^{abc}R^c + \frac{1}{N}\delta^{ab}, \tag{1.1}$$

where d^{abc} is totally symmetric. The action for pure $N=\frac{1}{2}$ supersymmetric gauge theory is given in components by[1]

$$\begin{aligned}S=&\frac{1}{2}\int d^4x\Big[-\tfrac{1}{2}F^{\mu\nu a}F^a_{\mu\nu}-2i\bar\lambda^a\bar\sigma^\mu(D_\mu\lambda)^a+D^aD^a-igC^{\mu\nu}d^{abc}F^a_{\mu\nu}\bar\lambda^b\bar\lambda^c\\&+\tfrac{1}{4}g^2|C|^2[d^{abe}d^{cde}(\bar\lambda^a\bar\lambda^b)(\bar\lambda^c\bar\lambda^d)+\tfrac{2}{N}h(\bar\lambda^a\bar\lambda^a)(\bar\lambda^b\bar\lambda^b)]\Big]\end{aligned}\tag{1.2}$$

with gauge coupling g, gauge field A_μ, gaugino λ and with

$$\begin{aligned}F^a_{\mu\nu}=&\partial_\mu A^a_\nu-\partial_\nu A^a_\mu-gf^{abc}A^b_\mu A^c_\nu,\\D_\mu\lambda^a=&\partial_\mu\lambda^a-gf^{abc}A^b_\mu\lambda^c,\end{aligned}\tag{1.3}$$

and $h=1$. (The $U(N)$ action would have a similar form to Eq. (1.2), except that the gauge indices would run over the extra $U(1)$ generator and we would have $h=0$.) In deriving Eq. (1.2) from the form given in Ref. [1] we are assuming that $\lambda=\lambda^aR^a$, $A_\mu=A^a_\mu R^a$ so that for instance $\mathrm{Tr}[(\bar\lambda\bar\lambda)^2]=\frac{1}{8}[d^{abe}d^{cde}(\bar\lambda^a\bar\lambda^b)(\bar\lambda^c\bar\lambda^d)+\frac{2}{N}(\bar\lambda^a\bar\lambda^a)(\bar\lambda^b\bar\lambda^b)]$; another choice of representation for λ, A_μ would simply result in a rescaling of the lagrangian. In Eq. (1.2), $C^{\mu\nu}$ is related to the non-anti-commutativity parameter $C^{\alpha\beta}$ by

$$C^{\mu\nu}=C^{\alpha\beta}\epsilon_{\beta\gamma}\sigma^{\mu\nu\gamma}_{\alpha},\tag{1.4}$$

where

$$\begin{aligned}\sigma^{\mu\nu}=&\tfrac{1}{4}(\sigma^\mu\bar\sigma^\nu-\sigma^\nu\bar\sigma^\mu),\\\bar\sigma^{\mu\nu}=&\tfrac{1}{4}(\bar\sigma^\mu\sigma^\nu-\bar\sigma^\nu\sigma^\mu),\end{aligned}\tag{1.5}$$

and

$$|C|^2=C^{\mu\nu}C_{\mu\nu}.\tag{1.6}$$

Our conventions are in accord with [1]; in particular,

$$\sigma^\mu\bar\sigma^\nu=-\eta^{\mu\nu}+2\sigma^{\mu\nu}.\tag{1.7}$$

Properties of C which follow from Eq. (1.4) are

$$\begin{aligned}C^{\mu\nu}\sigma_{\nu\alpha\dot\beta}&=C_\alpha{}^\gamma\sigma^\mu{}_{\gamma\dot\beta},\\C^{\mu\nu}\bar\sigma_\nu^{\dot\alpha\beta}&=-C^\beta{}_\gamma\bar\sigma^{\mu\dot\alpha\gamma}.\end{aligned}\tag{1.8}$$

The $N = \frac{1}{2}$ theory in components is manifestly invariant under the standard $SU(N)$ gauge transformations. Moreover Eq. (1.2) is invariant under the $N = \frac{1}{2}$ supersymmetry transformations (adapted to the $SU(N)$ case from Ref. [1])

$$\begin{aligned} \delta A_\mu^a =& - i\bar{\lambda}^a\bar{\sigma}_\mu\epsilon \\ \delta\lambda_\alpha^a =& i\epsilon_\alpha D^a + (\sigma^{\mu\nu}\epsilon)_\alpha \left[F_{\mu\nu}^a + \tfrac{1}{2}iC_{\mu\nu}d^{abc}\bar{\lambda}^b\bar{\lambda}^c\right] \\ \delta D^a =& - \epsilon\sigma^\mu D_\mu\bar{\lambda}^a. \end{aligned} \tag{1.9}$$

Whether the non-anticommutative superfield formalism may be similarly adapted for $SU(N)$ requires further investigation.[1]

2. One-loop calculation

We use the standard gauge-fixing term

$$S_{\rm gf} = \frac{1}{2\alpha}\int d^4x(\partial.A)^2 \tag{2.1}$$

with its associated ghost terms. The gauge propagator is

$$\Delta_{\mu\nu} = -\frac{1}{p^2}\left(\eta_{\mu\nu} + (\alpha - 1)\frac{p_\mu p_\nu}{p^2}\right) \tag{2.2}$$

and the gaugino propagator is

$$\Delta_{\alpha\dot{\alpha}} = \frac{p_\mu\sigma^\mu_{\alpha\dot{\alpha}}}{p^2}, \tag{2.3}$$

where the momentum enters at the end of the propagator with the undotted index. The one-loop graphs contributing to the "standard" terms in the lagrangian (those without a $C^{\mu\nu}$) are the same as in the ordinary $N = 1$ case, so gauge field and gaugino anomalous dimensions and gauge β-function are as for $N = 1$. Since our gauge-fixing term in Eq. (2.1) does not preserve supersymmetry, the gauge field and gaugino anomalous dimensions are different (and moreover gauge-parameter dependent). However, the gauge β-function is of course gauge-independent. The one-loop one-particle-irreducible (1PI) graphs contributing to the new terms (those containing C) are depicted in Figs. 1–3.

[1] We thank S. Terashima for a discussion on this point.

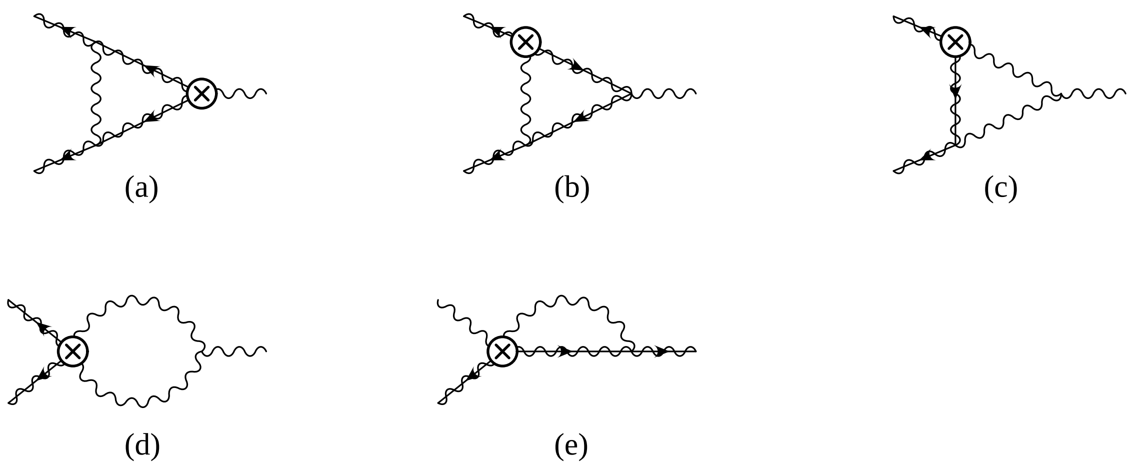

Fig.1: Diagrams with one gauge, two gaugino lines; the crossed circle represents the position of a C.

The divergent contributions to the effective action from the graphs in Fig. 1 are given by:

$$\begin{aligned}
\Gamma^{\text{pole}}_{1a} &= -\,(3+\alpha)\,NLT_1\\
\Gamma^{\text{pole}}_{1b} &= -NL\left(T_1+\tfrac{4}{3}A_1\right)\\
\Gamma^{\text{pole}}_{1c} &= -NL\left[\tfrac{1}{4}(2+7\alpha)T_1+\tfrac{1}{3}(2+3\alpha)A_1\right]\\
\Gamma^{\text{pole}}_{1d} &= \tfrac{1}{2}NL(5+\alpha)T_1\\
\Gamma^{\text{pole}}_{1e} &= NL\left[\tfrac{1}{4}(3-\alpha)T_1+(1+\alpha)A_1\right]
\end{aligned} \tag{2.4}$$

where

$$\begin{aligned}
T_1 &= id^{abc}gC^{\mu\nu}\partial_\mu A^a_\nu\bar\lambda^b\bar\lambda^c,\\
A_1 &= id^{abc}gC^\mu{}_\nu A^a_\mu\bar\lambda^b\bar\sigma^{\nu\rho}\partial_\rho\bar\lambda^c,
\end{aligned} \tag{2.5}$$

and (using dimensional regularisation with $d=4-\epsilon$) $L=\frac{g^2}{16\pi^2\epsilon}$.

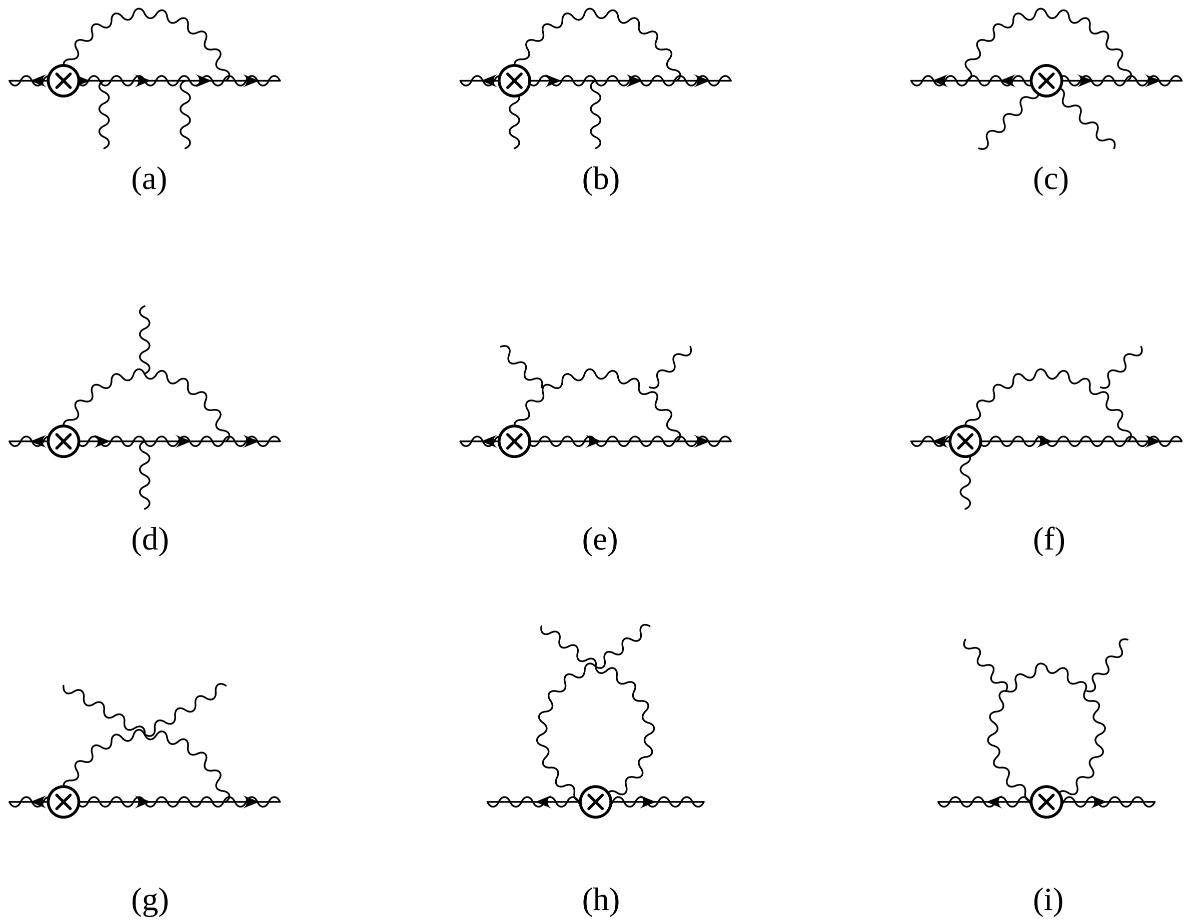

Fig.2: Diagrams with two gauge and two gaugino lines; the crossed circle represents the position of a C.

The results for Fig. 2 are given by:

$$\begin{aligned}
\Gamma_{2a}^{\text{pole}} &= NL\left(\tfrac{1}{2}T_2 + \tfrac{1}{3}A_2\right)\\
\Gamma_{2b}^{\text{pole}} &= 0\\
\Gamma_{2c}^{\text{pole}} &= \tfrac{1}{2}NL(3+\alpha)T_2\\
\Gamma_{2d}^{\text{pole}} &= 0\\
\Gamma_{2e}^{\text{pole}} &= NL\left[-\tfrac{1}{2}\alpha T_2 + \tfrac{1}{6}(4+3\alpha)A_2\right]\\
\Gamma_{2f}^{\text{pole}} &= NL\left[\tfrac{3}{4}\alpha T_2 - \tfrac{1}{2}(2+\alpha)A_2\right]\\
\Gamma_{2g}^{\text{pole}} &= NL\left(\tfrac{3}{4}\alpha T_2 + \tfrac{1}{2}A_2\right)\\
\Gamma_{2h}^{\text{pole}} &= -\tfrac{3}{4}NL(1+\alpha)T_2\\
\Gamma_{2i}^{\text{pole}} &= \tfrac{3}{4}\alpha NLT_2
\end{aligned} \tag{2.6}$$

where

$$
\begin{aligned}
T_2 &= id^{abe} f^{cde} g^2 C^{\mu\nu} A^c_\mu A^d_\nu \bar{\lambda}^a \bar{\lambda}^b, \\
A_2 &= id^{cde} f^{abe} g^2 C^{\mu\rho} A^c_\mu A^d_\nu \bar{\lambda}^a \bar{\sigma}^{\nu\rho} \bar{\lambda}^b.
\end{aligned}
\tag{2.7}
$$

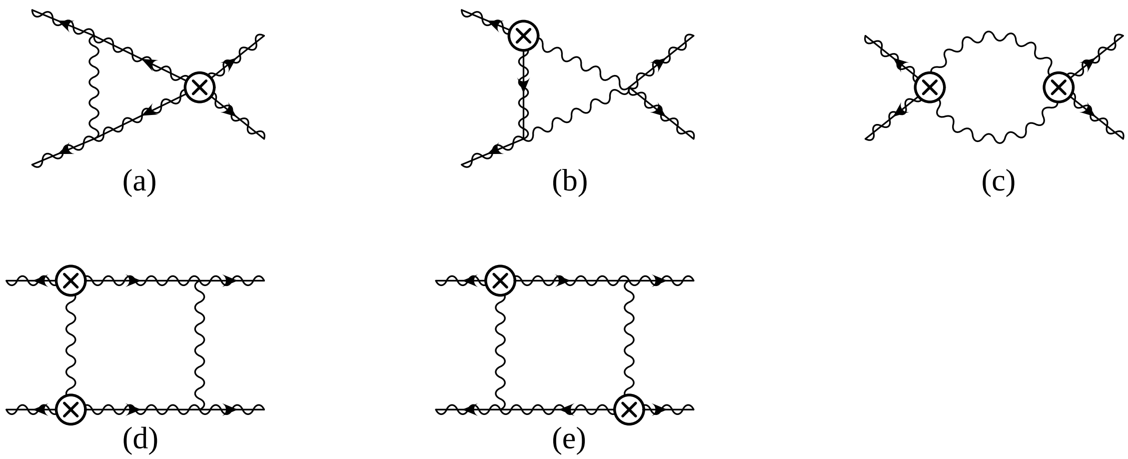

Fig.3: Diagrams with four gaugino lines; the crossed circle represents the position of a C or $|C|^2$.

Finally, the results for Fig. 3 are given by:

$$
\begin{aligned}
\Gamma^{\rm pole}_{3a} &= L\Big[\tfrac{1}{4}(3+\alpha)NT_3 + h(3+\alpha)\tilde{T}_3 - \tfrac{4}{N} f^{abe} f^{cde} h g^2 |C|^2 (\bar{\lambda}^a\bar{\lambda}^c)(\bar{\lambda}^b\bar{\lambda}^d) \\
&\quad + 2d^{abcd} g^2 |C|^2 [(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d) - \tfrac{1}{2}(\alpha-1)(\bar{\lambda}^a\bar{\lambda}^d)(\bar{\lambda}^c\bar{\lambda}^b)]\Big] \\
\Gamma^{\rm pole}_{3b} &= \tfrac{1}{2}\alpha NLT_3 \\
\Gamma^{\rm pole}_{3c} &= -\tfrac{1}{4}(1+\alpha)NLT_3 \\
\Gamma^{\rm pole}_{3d} &= -2d^{abcd} L g^2 |C|^2 [(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d) - \tfrac{1}{2}(\alpha-1)(\bar{\lambda}^a\bar{\lambda}^d)(\bar{\lambda}^c\bar{\lambda}^b)] \\
\Gamma^{\rm pole}_{3e} &= \tfrac{1}{3} f^{efa} f^{ghb} d^{gfc} d^{ehd} L g^2 |C|^2 [(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d) - (\bar{\lambda}^a\bar{\lambda}^d)(\bar{\lambda}^c\bar{\lambda}^b)]
\end{aligned}
\tag{2.8}
$$

where

$$
\begin{aligned}
T_3 =& d^{abe} d^{cde} g^2 |C|^2 (\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d), \\
\tilde{T}_3 =& g^2 |C|^2 (\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^b\bar{\lambda}^b), \\
d^{abcd} =& f^{iaj} f^{jbk} d^{kcl} d^{ldi}.
\end{aligned}
\tag{2.9}
$$

In obtaining these results we have made frequent use of the identity

$$
d^{iaj} f^{jbk} f^{kci} = -\frac{N}{2} d^{abc}. \tag{2.10}
$$

Other useful identities are

$$
\begin{aligned}
f^{abe} d^{ecd} + f^{ace} d^{edb} + f^{ade} d^{ebc} = 0, \\
f^{abe} f^{cde} = \tfrac{2}{N}(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}) + d^{ace} d^{bde} - d^{ade} d^{bce}.
\end{aligned}
\tag{2.11}
$$

Using[8]

$$f^{efa}f^{ghb}d^{gfc}d^{ehd} = \frac{N}{4}(d^{abf}d^{cdf} - d^{adf}d^{bcf} - d^{acf}d^{bdf}), \tag{2.12}$$

together with the Fierz identity

$$(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d) + (\bar{\lambda}^a\bar{\lambda}^c)(\bar{\lambda}^b\bar{\lambda}^d) + (\bar{\lambda}^a\bar{\lambda}^d)(\bar{\lambda}^b\bar{\lambda}^c) = 0 \tag{2.13}$$

we find

$$\Gamma_{3e}^{\text{pole}} = \tfrac{1}{4}NLT_3. \tag{2.14}$$

Then adding Eqs. (2.4), (2.6) and (2.8) we find the full result for the divergent contribution to the one-loop effective action from diagrams with one or two Cs:

$$\begin{aligned}\Gamma_{\text{1PI}}^{(1)\text{pole}} =& NL\int d^4x\Big[-\tfrac{5}{4}(1+2\alpha)T_1 + \tfrac{1}{4}(5+6\alpha)T_2 + \tfrac{1}{2}\left(\tfrac{3}{2}+\alpha-\tfrac{12}{N^2}h\right)T_3 \\ &+ \tfrac{h}{N}\left(3+\alpha-\tfrac{12}{N^2}\right)\tilde{T}_3 - A_1 + \tfrac{1}{2}A_2\Big].\end{aligned} \tag{2.15}$$

These results differ from those of Ref. [7] (even making the natural allowances for their use of the gauge group $U(N)$). The most obvious difference is that they claim that several diagrams, for which we obtain non-zero results, vanish when the diagrams with "crossed" fermion lines are taken into account. For instance, they claim that Figs. 1(c)–(e) vanish identically. This conclusion seems manifestly false; it is easiest to see this in the case of Fig. 1(d) where there are only external gaugino lines. Moreover the identity $C^{\rho\nu}\bar{\sigma}_{\nu}^{\dot{\beta}\delta}\sigma^{\kappa}_{\delta\dot{\delta}} = -C^{\rho\kappa}\delta^{\dot{\beta}}_{\dot{\delta}}$ (below their Eq. (45)) is simply not true.

At first sight, though, our results appear implausible; the terms A_1 and A_2 are clearly problematic since they violate gauge invariance. However, we have found that remarkably this difficulty can be resolved and the theory rendered renormalisable at a stroke by a divergent redefinition

$$\delta\lambda^a = -\tfrac{1}{2}NLgC^{\mu\nu}d^{abc}\sigma_{\mu}\bar{\lambda}^c A_{\nu}^b. \tag{2.16}$$

Note that this only affects the gaugino kinetic term. This results in a change in the action

$$\delta S = NL\int d^4x\left[-\tfrac{1}{4}(T_1 - T_2) + A_1 - \tfrac{1}{2}A_2\right]. \tag{2.17}$$

We therefore have

$$\begin{aligned}\Gamma_{\text{1PI}}^{(1)\text{pole}\prime} = \Gamma_{\text{1PI}}^{(1)\text{pole}} + \delta S =& NL\int d^4x\Big[-\tfrac{1}{2}(3+5\alpha)T_1 + \tfrac{3}{2}(1+\alpha)T_2 \\ &+ \tfrac{1}{2}\left(\tfrac{3}{2}+\alpha-\tfrac{12}{N^2}h\right)T_3 + \tfrac{h}{N}\left(3+\alpha-\tfrac{12}{N^2}\right)\tilde{T}_3.\Big].\end{aligned} \tag{2.18}$$

If we introduce bare fields and couplings according to

$$\begin{aligned} \bar{\lambda}_B = Z_\lambda^{\frac{1}{2}}\bar{\lambda}, \quad A_{B\mu} =& Z_A^{\frac{1}{2}}A_\mu, \quad g_B = Z_g g, \\ C_B = Z_C C, \quad |C|^2_B =& Z_{|C|^2}|C|^2, \quad h_B = Z_h h \end{aligned} \tag{2.19}$$

the C-dependent part of the bare action can be written as

$$\begin{aligned} S_B = \int d^4x \Big[& -Z_C Z_g Z_A^{\frac{1}{2}} Z_\lambda T_1 + \tfrac{1}{2} Z_C Z_g^2 Z_A Z_\lambda T_2 \\ & + \tfrac{1}{8} Z_{|C|^2} Z_g^2 Z_\lambda^2 T_3 + \tfrac{1}{4} Z_{|C|^2} Z_g^2 Z_\lambda^2 Z_h \tfrac{h}{N} \tilde{T}_3 \Big]. \end{aligned} \tag{2.20}$$

Of course a natural expectation would be that $Z_{|C|^2} = (Z_C)^2$ but this cannot be assumed as yet. As we mentioned before, the renormalisation constants for the fields and for the gauge coupling g are the same as in the ordinary $N = 1$ supersymmetric theory[5] and are therefore given by[9]:

$$\begin{aligned} Z_\lambda &= 1 - 2\alpha N L, \\ Z_A &= 1 + (3 - \alpha) N L \\ Z_g &= 1 - 3NL. \end{aligned} \tag{2.21}$$

Note that h is multiplicatively renormalised since the divergent term in $\tilde{T}_3$ is proportional to h, the coefficient of $\tilde{T}_3$ in the lagrangian Eq. (1.2); this is due to the cancellation of the d^{abcd} terms in Eq. (2.8) between $\Gamma_{3a}^{\text{pole}}$ and $\Gamma_{3d}^{\text{pole}}$ which otherwise would have produced non-homogeneous terms. This will be important shortly. We now find that if we take

$$\begin{aligned} Z_C = 1, \quad Z_{|C|^2} &= 1 + 48\tfrac{h}{N}L, \\ Z_h &= 1 - 6NL\left[1 + 8\frac{(h-1)}{N^2}\right], \end{aligned} \tag{2.22}$$

then

$$\Gamma^{(1)\text{pole}\prime} = \Gamma_{\text{1PI}}^{(1)\text{pole}\prime} + S_B^{(1)} = 0, \tag{2.23}$$

i.e. $\Gamma^{(1)\prime}$ is finite. So the form of the lagrangian is not quite preserved under renormalisation due to the non-zero h and the fact that $Z_h \neq 1$. As we remarked earlier, the original $N = \frac{1}{2}$ supersymmetric action[1] corresponds to taking $h = 1$ in Eq. (1.2). However because h is multiplicatively renormalised and $Z_{|C|^2} - 1 \propto h$, if we set $h = 0$ in Eq. (1.2) then we obtain a lagrangian whose form *is* exactly preserved under renormalisation–without even any need to renormalise the anticommutativity parameter C. The lagrangian with $h = 0$ is still $N = \frac{1}{2}$ supersymmetric and of course gauge-invariant.

3. Conclusions

We have shown how a combination of a minor modification to the pure $N = \frac{1}{2}$ supersymmetric action and a gauge-non-invariant divergent field redefinition lead to an action whose form is preserved under renormalisation at one loop. This is surprising because with this relatively simple action, $N = \frac{1}{2}$ supersymmetry imposes no constraints so we could have expected Z_C and $Z_{|C|^2}$ to be different and not equal to unity; (though with $Z_C \neq 1$, C would have to be replaced by C_B in the $N = \frac{1}{2}$ supersymmetry transformations). It will be interesting to see what happens in the case of $N = \frac{1}{2}$ supersymmetry including chiral matter, where there are several terms involving C and linked by $N = \frac{1}{2}$ supersymmetry. Work on this is in progress and we expect to report on it shortly. Of course it will also be worthwhile to see whether these properties persist at higher loops.

We have restored gauge invariance in this case by a somewhat unconventional expedient which works rather miraculously. One could speculate to what extent the $N = \frac{1}{2}$ supersymmetry and the identities Eq. (1.8) were required to make this trick work. It would be interesting to examine a theory of the same form but in which $C^{\mu\nu}$ was replaced by a general antisymmetric tensor. Moreover, suppose one considered a theory with an action based on Eq. (1.2) but including all the hermitian conjugate terms which are missing. The only new diagrams would simply be the "hermitian conjugates" of those in Figs. 1–3. Eq. (2.16) would now need to be supplemented by its hermitian conjugate. However, the variation of the action would now include additional unwanted non-gauge-invariant terms since it is now not only the gaugino kinetic term which varies. This raises the spectre of a theory (albeit non-renormalisable) with ineradicable non-gauge-invariant divergences.

We have adapted the $N = \frac{1}{2}$ action in components from the gauge group $U(N)$ to $SU(N)$, partly because this simplifies the calculations. On the other hand, from our current perspective $U(N)$ presents the *prima facie* advantage that the $\tilde{T}_3$ term would not appear explicitly in Eq. (1.2) and one might hope that Eq. (1.2) might be form-invariant under renormalisation as it stands. However, the $U(N)$ version of Eq. (2.12) includes extra terms[10] which lead to $\tilde{T}_3$ being generated despite not being present in Eq. (1.2)–so the form-invariance is inescapably lost in this case. Moreover, while the $SU(N)$ gauge coupling renormalises in accordance with Eq. (2.21), the $U(1)$ coupling would of course be unrenormalised.

Finally we do not as yet have a theoretical justification or interpretation for the field redefinition which we appear to be compelled to make, and this point deserves investigation.

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